SOLUTIONS FOR PROBLEMS 1 - 30

1. **Answer: 5**
Evaluate $-x^2 - 3x + 9$ for $x = 1$

When substituting $x = 1$ in $-x^2$ be sure to do the exponent before the multiplication by –1 to get $-(1)^2 = -1$. $-1 - 3 + 9 = 5$

2. **Answer:** $6\sqrt{3}+4$ When multiplying $\sqrt{3}\left(\sqrt{3}\right)=3$ so that $\sqrt{3}\left(\sqrt{3}+7\right)-\left(\sqrt{3}-1\right)$ becomes $3+7\sqrt{3}-\sqrt{3}+1=6\sqrt{3}+4$

3. **Answer: 2(3*x*-*y*)** Using the distributive law
$-2(y - x) = -2y + 2x$ we get $2x - 2y$

$$\begin{aligned} & 5x - 2(y - x) - x \\ &= 5x - 2y + 2x - x \\ &= 6x - 2y \\ &= 2(3x - y) \end{aligned}$$

4. **Answer:** $\frac{1}{4}$ $\quad \frac{\frac{3}{4}}{3} = \frac{3}{4}\left(\frac{1}{3}\right) = \frac{1}{4}$

5. **Answer:** $\frac{21}{22}$ Write the numerator as a single fraction with denominator of 4,

$1+\frac{3}{4}=\frac{4}{4}+\frac{3}{4}=\frac{7}{4}$ and write the denominator as a single fraction with denominator of 6,

$2-\frac{1}{6}=\frac{2(6)}{1(6)}-\frac{1}{6}=\frac{12}{6}-\frac{1}{6}=\frac{11}{6}$ Now divide $\frac{7}{4}$ by $\frac{11}{6}$. Be sure to invert and multiply to get $\frac{7}{4}\left(\frac{6}{11}\right)=\frac{7}{2}\left(\frac{3}{11}\right)=\frac{21}{22}$. Here we divided the denominator 4 and the numerator 6 by 2.

6. **Answer:** $6x^4y^2\sqrt{2}$

$\sqrt{72}=\sqrt{36(2)}=6\sqrt{2}$.

$\sqrt{x^8}=\sqrt{x^4(x^4)}=x^4$ since x^4 is positive. Similarly $\sqrt{y^4}=y^2$

7. **Answer:** $\frac{2\sqrt{6}}{3}$

Since $\sqrt{6}\left(\sqrt{6}\right) = 6$, we write $\frac{4}{\sqrt{6}} = \frac{4}{\sqrt{6}}\left(\frac{\sqrt{6}}{\sqrt{6}}\right) = \frac{4\sqrt{6}}{6} = \frac{2\sqrt{6}}{3}$

8. Answer: $-24x^5y^8$

When multiplying exponent expressions with the same base, keep the base and add the exponents thus, $x^3(x^2) = x^5$ and $y(y^7) = y^8$

9. **Answer:** $\frac{2}{3}$ Write the equation of the line in the form $y = mx + b$ where m is the slope. $3y = 2x - 10$, $y = \frac{2}{3}x - \frac{10}{3}$

10. **Answer:** $\frac{7\sqrt{2}}{2}$

Write $\sqrt{18} = \sqrt{9(2)} = 3\sqrt{2}$ and rationalize $\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \frac{\sqrt{2}}{2}$

Now we are adding $3\sqrt{2} + \frac{\sqrt{2}}{2} = \frac{2\left(3\sqrt{2}\right)}{2} + \frac{\sqrt{2}}{2} = \frac{6\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \frac{7\sqrt{2}}{2}$

11. Answer: $x = \frac{5}{2}$

Multiply both sides of the equation by the denominator $x - 3$

$$(x-3)\left[\frac{1}{x-3} - 3\right] = (x-3)\left[\frac{x}{x-3}\right]$$

Use the fact that $(x-3)\left(\frac{1}{x-3}\right) = 1$

$1 - 3(x - 3) = x$

$1 - 3x + 9 = x$

$10 = 4x$

$x = \frac{10}{4} = \frac{5}{2}$

12. Answer: : $x = -5$, $x = 5$ There are two values of whose absolute value is $|-5| = 5$ and $|5| = 5$.

13. Answer: $-\dfrac{1}{3(x+3)}$ We factor $x^2 - 3x = x(x - 3)$ and $9 - x^2 = (3 - x)(3 + x)$

Remember the difference of squares factorization $a^2 - b^2 = (a - b)(a + b)$

Thus we get $\left(\dfrac{x^2-3x}{3x}\right)\left(\dfrac{1}{9-x^2}\right)=\dfrac{x(x-3)}{3x}\left[\dfrac{1}{(3-x)(3+x)}\right]=\dfrac{x(x-3)}{3x(3-x)(3+x)}$

Use the fact that $x - 3 = -(3 - x)$, so that $\dfrac{x-3}{3-x}=-1$

Putting this together we get $=-1\left[\dfrac{1}{3(3+x)}\right]=-\dfrac{1}{3(3+x)}$

14. Answer: $x < -6$ $2x + 1 > 3x + 7$

Subtract $3x$ from both sides $-x + 1 > 7$

Subtract 1 from both sides $-x > 6$

Multiply by -1 $x < -6$

Remember multiplying an inequality by a minus changes the sense of the arrow.

15. Answer $x=\dfrac{1}{2}$ and $x = -2$

Consider the standard quadratic form $ax^2 + bx + c = 0$

Whose solution is $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

For the solution of $2x^2 + 3x - 2 = 0$, $a = 2$, $b = 3$, and $c = -2$.

Since the discriminant $b^2 - 4ac = 3^2 - 4(2)(-2) = 9 + 16 = 25$
is a perfect square, we can factor directly:

$2x^2 + 3x - 2 = 0$,

$(2x - 1)(x + 2) = 0$

$2x - 1 = 0$ $x + 2 = 0$

$x=\dfrac{1}{2}$ $x = -2$

16. Answer: $\dfrac{3 \pm \sqrt{41}}{4}$

For the solution of $2x^2 - 3x - 4 = 0$, we have $a = 2$, $b = -3$, $c = -4$. The discriminant $b^2 - 4ac = (-3)^2 - 4(2)(-4) = 9 + 32 = 41$ is not a perfect square so that we must use the quadratic formula and get two real roots:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Thus $x = \dfrac{-(-3) \pm \sqrt{(-3)^2 - 4(-3)(-4)}}{2(2)} = \dfrac{3 \pm \sqrt{9+32}}{4} = \dfrac{3 \pm \sqrt{41}}{4}$

17. Answer: $\dfrac{3 \pm i}{2}$

For the solution of $2x^2 - 6x + 5 = 0$, $a = 2$, $b = -6$, $c = 5$

The discriminant $b^2 - 4ac = (-6)^2 - 4(2)(5) = 36 - 40 = -4$. Since the discriminant is negative, the two roots are imaginary.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(6) \pm \sqrt{(-6)^2 - 4(2)(5)}}{2(2)} = \frac{6 \pm \sqrt{36-40}}{4} = \frac{6 \pm \sqrt{-4}}{4}$$

$$x = \frac{6 \pm \sqrt{4(-1)}}{4} = \frac{6 \pm \sqrt{4}\sqrt{-1}}{4} = \frac{6 \pm 2i}{4}$$

Thus, here we use $i = \sqrt{-1}$

We can reduce the answer by factoring 2 in the numerator

$$\frac{6 \pm 2i}{4} = \frac{2(3 \pm i)}{4} = \frac{3 \pm i}{2}$$

18. Answer: $\dfrac{9}{4}$

The negative exponent means we have to take the reciprocal of what is in the parentheses and then square. $\left(\dfrac{2}{3}\right)^{-2} = \left(\dfrac{3}{2}\right)^{2} = \dfrac{9}{4}$

Remember when raising a fraction to a power, both the denominator and numerator are raised to that power

19. Answer: $x = 3$, $y = 2$
Solve for x and y there are two methods that are often used.

Addition method: Multiply the second equation by – 5

$5x - 3y = 21$

$-5x - 25y = 35$

Add the equations to get:

$-28y = 56$

$$y = -\frac{56}{28} = -2$$

Substitute back into the second equation to get:

$x + 5(-2) = -7$

$x - 10 = -7$

$x = -7 + 10 = 3$

Thus $x = 3$, $y = 2$.

Substitution method: Solve for x in the second equation to get:

$x = -7 - 5y$

Substitute for x in the first equation to get:

$5(-7 - 5y) - 3y = 21$

$-35 - 25y - 3y = 21$

$-35 - 28y = 21$

Add 35 $\quad -28y = 21 + 35$

$-28y = 56$

Divide by $\quad -28$

$$y = -\frac{56}{28} = 2$$

Substitute in to get x, substitute the value $y = -2$ in $x = -7 - 5y$ to get
$x = -7 - 5(-2) = -7 + 10 = 3$

Thus $x = 3$, $y = 2$.

20. Answer: $3 < x < 7$

We can do the problem algebraically or geometrically.

Algebraically $|x-2| < 5$ means $-5 < x - 2 < 5$

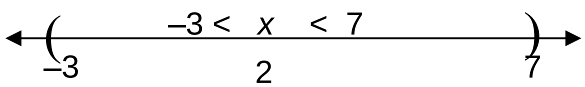
So by adding to all three parts of the inequality we get

$$-5 < x - 2 < 5$$

$$-5 < x - 2 + 2 < 5 + 2$$

$$-3 < x < 7$$

Geometrically $|x-2| < 5$ means that the distance (in both directions) from x to 2 is less than 5. So if we move 5 units to the right (up the axis) from 2 we get 7 and 5 units to the left (down the axis) from 2 we get –3



21. Answer: $\frac{3y-4x}{xy}$

The least common denominator is xy.

Thus $\frac{3}{x} - \frac{4}{y} = \frac{3y}{xy} - \frac{4x}{xy}$.

Since the denominators of the two fractions are now the same we can add the numerators getting

$$\frac{3}{x} - \frac{4}{y} = \frac{3y}{xy} - \frac{4x}{xy} = \frac{3y-4x}{xy}.$$

22. Answer: $(x-3)(x+3)(x^2+9)$

Recall the factorization of the **difference of squares** $a^2 - b^2 = (a-b)(a+b)$

$x^4 - 81$ is a difference of squares namely $(x^2)^2$ and 9^2.

$$x^4 - 81 = (x^2 - 9)(x^2 + 9).$$

The first factor is again a difference of squares:

$$(x^2 - 9) = (x-3)(x+3)$$

$$x^4 - 81 = (x^2 - 9)(x^2 + 9) = (x-3)(x+3)(x^2+9)$$

23. Answer: $\dfrac{11x}{24y}$

The least common denominator is $24y$.

Thus $\dfrac{x}{8y}+\dfrac{x}{3y}=\dfrac{3x}{24y}+\dfrac{8x}{24y}=\dfrac{11x}{24y}$.

Here since the denominators are the same we add the numerator.

24. Answer: The lines are parallel.

If you solve for y in each equation to get the form $y = mx + b$, you can examine the slopes m and the y intercepts b.

Equation 1 becomes: $y=-\dfrac{3}{4}x-\dfrac{7}{4}$

Equation 2 becomes: $y=-\dfrac{9}{12}x-\dfrac{2}{12}$

which reduces to $y=-\dfrac{3}{4}x-\dfrac{1}{6}$

Since the slopes are equal and the y intercepts are not, the lines are parallel.
In the case that both the slopes and the y intercepts were equal, the lines would be the same. In the case that the slopes are unequal, the lines intersect in one point.

25. Aswer: 2

Think of as $(8)^{\frac{2}{3}}$ as $\left(8^{\frac{1}{3}}\right)^2$

$8^{\frac{1}{3}}$ is the cube root of 8 often written as $\sqrt[3]{8}=2$ So that $\left(8^{\frac{1}{3}}\right)^2=2^2=4$

$(4)^{-\frac{1}{2}}=\dfrac{1}{4^{\frac{1}{2}}}$, the meaning of a negative exponent.

$4^{\frac{1}{2}}$ is the square root of 4

often written $\sqrt{4}=2$.

Thus $(8)^{\frac{2}{3}}(4)^{-\frac{1}{2}}=4\left(\dfrac{1}{2}\right)=2$

26. Answer: $x < 2$ or $x > 3$

$$x^2 - 5x + 6 > 0$$

To find the solution set for *x* set the inequality equal to zero and factor and solve for *x*:

$$x^2 - 5x + 6 = 0$$
$$(x-3)(x-2) = 0$$
$$x = 3, \quad x = 2$$

Since the inequality is strictly greater than zero, neither of these are in the solution set. These two numbers divide the *x* axis into three sections:
$-\infty < x < 2$, $2 < x < 3$, and $3 < x < \infty$
Pick any "test" number for each section and substitute into the inequality:

For $-\infty < x < 2$, say $x = 0$.
Substituting $x = 0$ into $x^2 - 5x + 6$ gives us $(0)^2 - 5(0) + 6 =$ **6** which is greater than zero.
Thus $-\infty < x < 2$ is part of the solution set.

For $2 < x < 3$, we can test with $x = 2\frac{1}{2}$ or $x = \frac{5}{2}$

Substituting $x = \frac{5}{2}$ into $x^2 - 5x + 6$ gives us $-\frac{1}{4}$ which is less than zero.

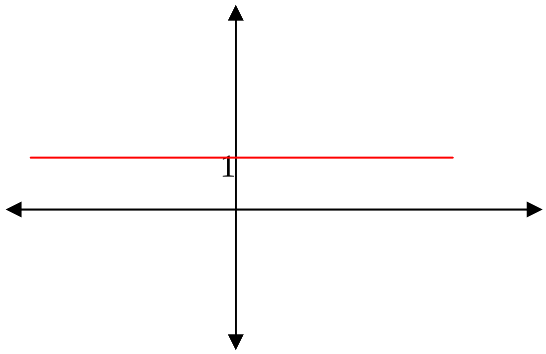
$$\left(\frac{5}{2}\right)^2 - 5\left(\frac{5}{2}\right) + 6 = \frac{25}{4} - \frac{25}{2} + 6 = \frac{25 - 25(2) + 6(4)}{4} = \frac{25 - 50 + 24}{4} = \frac{49 - 50}{4} = -\frac{1}{4} < 0$$

Thus $2 < x < 3$, is NOT in our solution set.
For $3 < x < \infty$, we can substitute $x = 4$ to get $4^2 - 5(4) + 6 = 16 - 20 + 6 = 12 > 0$.
Thus $3 < x < \infty$ is part of the solution set.

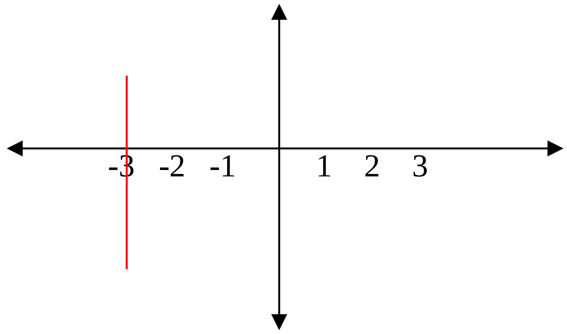
Thus the complete solution set is $\{x \mid -\infty < x < 2$ **or** $3 < x < \infty\}$ which can also be written in interval notation as $(-\infty, 2) \cup (3, \infty)$.

27. Answer: 28
If $f(x) = x^3 + 1$, $f(3) = 3^3 + 1 = 28$

28. Answer: The horizontal line one unit above the *x* axis.



29Answer: The vertical line 3 units to the left of the *y* axis.



30. Answer:

To graph $2x + 3y = 2$, we need to find any two points which lie on the line and connect them with the straight edge. There are several ways to do this. We outline two common methods.

Intercept method: If we let $x = 0$, the *y* intercept is $3y = 2$ or $y = \frac{2}{3}$ and if we let $y = 0$ the *x* intercept is (1, 0).

Slope-Intercept method: Solving for *y* in the form $y = mx + b$ we get: $y = -\frac{2}{3}x + \frac{2}{3}$ From this form we see that the *y* intercept is $\frac{2}{3}$. From this point we can use the slope, which is $-\frac{2}{3}$, to find a second point by moving, 2 units to the left (0 – 2 = - 2) and 3 units up $\left(\frac{2}{3} + 3 = \frac{11}{3}\right)$ to get the second point $\left(-2, \frac{11}{3}\right)$

PICTURE